Table 1 Data input summary ($\mu_e = 21$; $M_e = 3.98$; $T_{t_e} = 5670°R$; $\gamma = 1.25$; $r_e = 0.398$ ft)

Case	p_0, mbar	T_0, °R	μ_0	P_e, psia	p_e/p_0	x_1/r_e
1	20	360	32	0.330	1.14	24.4
2	5	495	31	0.179	2.46	42.6
3	20	360	32	0.179	0.613	14.9

profiles at station x. For the v and T_t profile we use[1]

$$(v/v_c)_x = [1 + (y/y_b)^2]_x{}^{3/2} \tag{12}$$

$$(\Delta T_t/\Delta T_{t_c})_x = 1 - (y/y_b)_x{}^{3/2} \tag{13}$$

This procedure is limited to an isobaric jet, since Eq. (11) [deduced from Eqs. (3) and (4)] is based on this assumption. The effect of nonuniform pressure is introduced in Eq. (11) when one substitutes for $\bar{v}_c$ the values obtained for the slightly underexpanded jet given by Eq. (10).

If the physical growth of the plume is desired, Eq. (7) can now be integrated to yield $y_b(z)$. Furthermore, Eqs. (12) and (13) provide sufficient information to determine all the flow-field variables throughout the jet.

A numerical calculation along the foregoing lines is feasible, but if, following the philosophy of this exposition, one is interested primarily in a simple method for reasonable estimates of the plume interaction with the atmosphere, one can take advantage of the fact that A_xD_e/B_eC_x does not vary significantly for different choices of profiles and can be taken as 0.745. The following calculations have been done under this assumption; thus, one does not need to know the internal flow in the nozzle, but only the average values of M_e, p_e, and T_{t_e}.

The preceding analysis was programed in Fortran IV for the IBM 360 computer. Three cases were computed as typical of Mars landing requirements; the input parameters are listed in Table 1. For each case the following constraints were imposed: the descent to Mars was along a local vertical to a smooth plane surface. There was no atmospheric wind present, and the exhaust gases were invariant in composition.

The results of these computations are summarized in Figs. 1 and 2. From these results one may obtain the desired boundaries for which a significant effect of the exhaust jet is experienced, e.g., $p_{t_c}' = 1.1\ p_0$, and $T_t = T_0 + 10°R$. These effects are not significant within the range of axial locations shown in the figures; however, the locations for this T_t effect are 3200, 3900, and 2300 exit radii, respectively. In each case the significant rise in surface pressure will occur before the rise in temperature.

The assumptions involved in these results should be reiterated. In using Eq. (6) for the propagation of a jet, an incompressible law has been extended to a supersonic flow. The validity of this extension has been confirmed by experiments[5] for specific types of jets. Also in this law, the parameter β has been established for flows similar to those considered herein to be 0.22, and the function of v_{ch} used in this equation is a relatively simple expression which has been used to facilitate the mathematics. Better approximations for v_{ch} with the attendant complications to the mathematics are known to exist but were not used in this approximate analysis. Finally, in this equation for the growth of the jet, it was assumed that $\mu_e = \mu_0$. To extend the technique to nonequal molecular weights, another equation would be required. (However, $\mu_e/\mu_0 = 21/31 = 0.7$ should not have a strong effect.)

The profiles of velocity and stagnation temperature assumed in this writing have been confirmed by experiments.[1,5] It is important to note that these profiles will affect most significantly the distribution of Mach numbers. Since the distributions of other variables enter through integrals in the determination of A_x, B_e, C_x, and D_e, these coefficients are not very sensitive to the exact profiles.

Although $p_{t_c}'(x)$ is computed from normal shock relations and thus is accurate, the transport of energy due to turbulence is not considered in this calculation. However, it is felt that the turbulence transport is sufficiently small at large downstream locations that the inviscid solutions to the shock problem are adequate.

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Hele-Shaw and Porous Medium Flow for Space Fuel Cells

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THIS Note presents an investigation of two types of flow which may eliminate flow problems in certain space missions. The two flow systems which have been investigated are very similar, and consist of fuel tanks filled with porous, medium, or Hele-Shaw cells. (The Hele-Shaw cells are basically thin parallel membranes or plates placed close together in a fuel tank.) The main changes these devices produce are as follows: 1) the inertia forces are reduced due to the increased viscous action of the surfaces; 2) the Bond number takes on a value less than one, and sloshing is eliminated as a problem; 3) surface tension forces act over a small local region, although they increase; 4) funnelling can be more readily controlled by adjusting the porous medium permeability or Hele-Shaw plate spacing at the fuel tank outlet; and 5) an interface instability may develop at the interface of the liquid fuel and driver gas or liquid.

The experimental results that were obtained came from the use of a Hele-Shaw channel, and the porous medium behavior was inferred from an analogy which exists between the two types of flow. This analogy can be shown quite readily by comparing the functional form of the equation for the mean velocity across a Hele-Shaw channel with the average velocity in a porous medium. The average velocity, **u**, across a Hele-Shaw channel is given by[1]

$$\mathbf{u} = -b^2\nabla P/(12\mu) \tag{1}$$

where b is the distance between the parallel plates, μ the fluid viscosity, ∇P the gradient of pressure, and **u** is the velocity vector. For flow in a porous medium the average velocity is given by Darcey's law as

$$\mathbf{u} = k\nabla P/\mu \tag{2}$$

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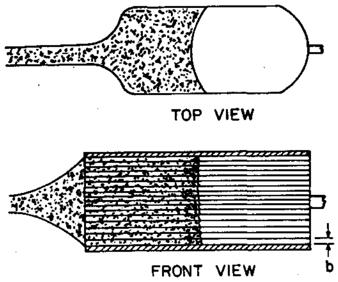

Fig. 1 Hele-Shaw fuel tank.

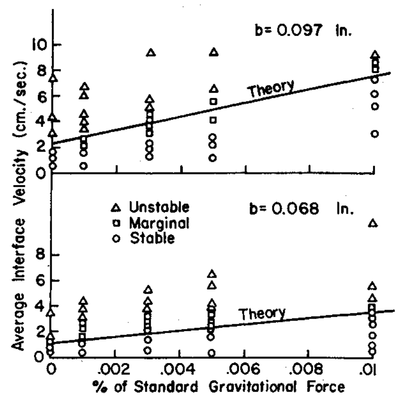

Fig. 3 Hele-Shaw stability results.

where k is the permeability of the porous medium. By comparing Eqs. (1) and (2) it can be seen that if $b^2/12 = k$, the expressions for the average velocities are identical. Also, since both flow systems must satisfy an average continuity equation, the differential equation for the pressure is the same (Laplace's Eq.). Therefore, if the Hele-Shaw cell and porous medium tank are given the same geometry and the same boundary conditions on the pressure, a formal analogy will exist between the two flow systems. (Since the Hele-Shaw flow is limited to two dimensions, the analogy will hold quantitatively only for two-dimensional porous medium flow, and it will not strictly represent the behavior of axisymmetric porous medium tanks.)

Figure 1 depicts a possible Hele-Shaw fuel tank. Membranes or plates divide the flow into a series of Hele-Shaw channels, which reduce the characteristic flow length and increase greatly the importance of viscous to inertia forces. Figure 1 also shows the interface between the driver gas and the liquid fuel. An important factor in the behavior of this system will be the stability of the gas-liquid interface. Saffman and Taylor[2] showed that a sufficient condition for a stable interface to exist between two fluids such that small displacements of the interface do not grow into large penetrations of the driver fluid into the driven fluid is

$$24\pi V(\mu_1 - \mu_2)/lb^2 + (\rho_1 - \rho_2)g - 8\pi^3 T/l^3 < 0 \quad (3)$$

where subscripts 1 and 2 refer to the driven and driving fluids respectively, V the interface velocity, ρ the density, T the interface surface tension, g acceleration or gravitational force directed from 1 to 2, and l the width of the Hele-Shaw channel. For most fuel systems the driven fluid is a liquid while the driver is a gas. Under these conditions it is usually true that $\mu_1 > \mu_2$ and $\rho_1 > \rho_2$. Therefore, with zero-gravity conditions the surface tension forces stabilize the flow, while viscous forces tend to destabilize the flow. It can also be seen from Eq. (3) that, if the velocity is large enough, the flow will always be unstable.

Equation (3) does not strictly hold for a porous medium, because the basic behavior of the surface tension force is different at the interface in a porous medium as compared to a Hele-Shaw cell. In the Hele-Shaw cell the interface acts somewhat like a continuous flexible membrane, while this membrane is broken into discontinuous parts for porous medium flow. Therefore, it is to be expected that a porous medium tank may be more unstable than an equivalent Hele-Shaw tank, because of the behavior of the surface tension force. It is difficult to prove this analytically because of the complex nature of a gas-liquid interface in a porous medium.

Apparatus and Experimental Procedure

The equipment for the investigation consisted of two 20 × 30 in. pieces of commercial plate glass separated by a rubber gasket of desired shape and thickness (see Fig. 2). In order to level the system or impose a specified slope (i.e., to regulate the gravitational force) a three screw leveling system was used. The desired pressure gradient to provide interface movement was obtained by raising or lowering a large plastic bottle connected by tygon tubing to the outlet nozzle at the end of the glass plates. The flow rate was kept constant by having the majority of the pressure drop occur across a valve in the tygon tubing. (This is basically the same type of system as Saffman and Taylor).[2]

Two basic tank shapes were studied in the investigation, a rectangular and a cylindrical tank, however, only the results of the rectangular tank study will be presented (the spherical tank showed essentially the same behavior as the rectangular but had a greater tendency toward funnelling). Air was used as the driving gas and water as the fuel and with the two fluids specified there are four variables that determine stability. These are the velocity **u**, the spacing b, the critical length l, and the gravitational force g. The experimental procedure

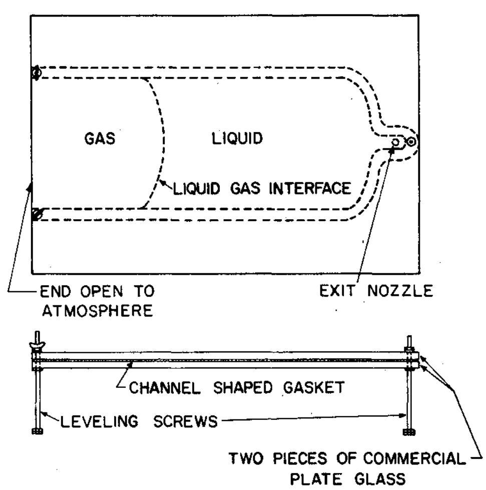

Fig. 2 Experimental apparatus.

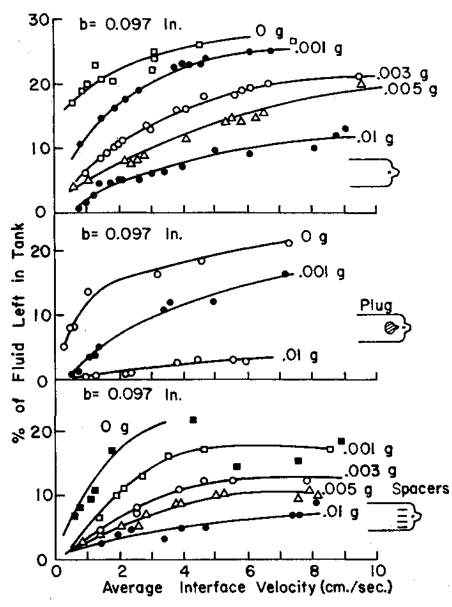

Fig. 4 Outlet funnelling results.

was to eliminate b and l as variables by choosing a particular geometry. The gravitational force was then varied and the critical velocity $\mathbf{u}$ was found experimentally. Only favorable gravitational forces (those directed from the gas to the liquid) were used since unfavorable gravitational forces tend to dominate the stability equation. In practical situations the pressure gradient is usually favorable during the firing of the rocket when fuel is being used.

Experimental Results

Shown in Fig. 3 are the results of the investigation of the stability of the interface in the rectangular channel for two different plate spacings. The average interface velocity was obtained by measuring the distance and time required for the gas-liquid interface to move from the starting position to the tank outlet. The solid lines represent the predictions of Eq. (3) for the conditions of the experiment and for an infinite tank. The three states of the interface have been labeled stable, neutral, and unstable. The stable interface condition existed when the interface remained flat with very little penetration of the gas into the liquid, while the neutral case had moderate penetration of the gas. The unstable case represents large penetration of fingers[2] into the liquid fuel.

Agreement of the data points and theory is seen to be good. The major difference is due to the large neutral region, and is the result of funnelling pressure gradients caused by the finite tank geometry. The neutral region was larger for the low-gravity tests where funnelling would be more influential. For the tests with higher gravitational forces (0.01 g) the experimental results were dominated by the gravitational forces, and the region of neutral stability became very small. Changing the plate spacing b did not cause any observable qualitative changes in the comparison between theory and experiment, and the correct quantitative changes were predicted by Eq. (3). All the tests were carried out at a creeping flow Reynolds numbers[1] for which the Hele-Shaw assumptions were justified.

The major source of experimental error was due to small inaccuracies in the leveling of the channel in the direction transverse to the flow. This error resulted in a tendency of the fluid to flow slightly down one side of the channel more than the other. The maximum magnitude of the transverse gravitational field at any point in the channel was 0.001 g, and the influence of this field was the greatest for the very low-gravity tests.

Another important characteristic of a fuel tank system is the amount of fluid left in the tank after the driver gas reaches the exit. Figure 4 shows the results of the measurement of the percentage of fluid left remaining in the system. The tests were carried out with three outlet geometries in order to determine the influence of funnelling on the data. These outlet geometries consisted of the following: a) no exit constriction; b) single row of spacers in the exit; and c) a large rounded plug in the exit. It was found that both the spacers and the plug improved the performance of the Hele-Shaw cell. The greatest improvement occurred with the large plug in the exit and this indicates that funnelling can be reduced, since the plug changes the exit pressure distribution. The exit spacers also reduced the amount of fluid left in the tank, and this was a result of increased surface tension in the exit. For low velocity ($V > 2$ cm/sec) and moderate gravitational force ($g > 0.005$) it was found that the percent of fluid left in the tank could be kept under 5%, which is a respectable result.

Conclusions

The major conclusions of this investigation are the following: 1) Hele-Shaw and porous medium fuel tank flow systems are feasible, and may represent an effective way of controlling sloshing in low gravity environments; 2) funnelling may be reduced considerably by changing the outlet drain configuration of the Hele-Shaw channel; 3) almost full recovery of the liquid fuel may be obtained by keeping the interface velocity below the interface stability limit; 4) the influence of surface tension is to stabilize the gas-liquid interface in the Hele-Shaw channel. It is not yet clear how surface tension will influence porous medium tanks.

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Leading Characteristic in Conical Nozzle Flows and the Effect of Manufacturing Errors

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CONICAL expansion nozzles followed by contoured nozzles are of use in any supersonic wind tunnel.[1] These shapes are attractive because of the availability of the nozzle contours developed by Cresci[2] and because of their relative ease of manufacture. In these nozzle contours it is assumed that a relatively long conical section (long with respect to the throat radius) has fully conical inviscid flow at its exit. Then a contour is generated using the theory of characteristics which will produce a final uniform Mach number and parallel flow.

It has been observed by Edenfield[1] that slight manufacturing errors produce relatively large overexpansions at the beginning of the recompression region, even when the influence of the boundary layer displacement thickness was accounted for. The displacement thickness correction was everywhere less than 3% of the local radius and was accounted for in the nozzle contour. The accuracy of displacement thickness prediction was approximately 10% or within 0.3% of the nozzle radius. Several nozzles were reported with progressively better allowances for displacement thickness. The purpose of this Note is to provide an estimate of effects of manufacturing errors on the overexpansion.

Along a characteristic ξ, we can write (see Fig. 1)

$$d\xi \cos(\mu - \theta) = dx$$

and

$$-d\xi \sin(\mu - \theta) = dy \tag{1}$$

where dx and dy are the changes in x and y along the ξ characteristic for a change in ξ of $d\xi$, μ is the Mach angle and θ is the local flow inclination to the axis. Then we can write

$$dy/dx = -\sin(\mu - \theta)/\cos(\mu - \theta) \tag{2}$$

where dy/dx is the slope of the ξ characteristic. For the hypersonic case in which μ and θ are both small,

$$dy/dx \simeq -(\mu - \theta) \tag{3}$$

where $\tan\theta \simeq \theta \simeq y/x$ for small θ and

$$\mu = \sin^{-1}(1/M) \simeq 1/M \text{ or } \mu \simeq c/u$$

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